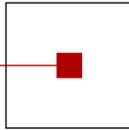


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# Advances in Knowledge-Based Technologies

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## **Program**

**Chair: Susanne Saminger-Platz**

13:00 Bernhard Moser:

The Range of a Simple Random Walk on  $\mathbb{Z}$ : An Elementary Combinatorial Approach

13:30 Ayadi Chouikhi:

Image Segmentation by Chan-Vese Method

14:00 Edwin Lughofer:

Dynamic Inclusion of New Event Types in Visual Inspection using Evolving Classifiers



# The Range of a Simple Random Walk on $\mathbb{Z}$ : An Elementary Combinatorial Approach

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January 30, 2015

## Abstract

This presentation reviews the article [7]. The problem of determining the distribution of the range of a simple random walk has been treated extensively in the literature. Feller [3] computes the distribution of the range of a standard Brownian motion and derives estimates for the discrete case. In this article he points out that the problem of finding exact formulae for the distribution of the range is difficult to solve in the discrete case. The asymptotic behaviour of the range is investigated, e.g., by [2], [4], [9]. In 1996 the problem was solved by P. Vallois by exploiting martingale techniques [10]. However, the solution by elementary combinatorics has remained an open problem since 1951.

In this research we demonstrate that Feller's problem can be solved in an elementary and concise manner. We present two approaches, both of them rely on Hermann Weyl's discrepancy measure [11]. The first approach exploits the fact that the range of the partial sums of the elements of a sequence defines a norm, the discrepancy norm. The  $n$ -dimensional unit balls of this norm can be characterized as a zonotope. This allows us to turn the original combinatorial problem on  $\mathbb{Z}$  into a known path enumeration problem on a bounded lattice  $L_d = (0, 1, \dots, d)$ . The solution is expressed in terms of the adjacency matrix  $\mathbf{Q}_d$  of the corresponding bounded walk. The

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second approach exploits the algebraic structure of the adjacency matrix  $\mathbf{Q}_d$  by representing it as sum of a left and a right shift matrix,  $\mathbf{Q}_d^-$  and  $\mathbf{Q}_d^+$ , respectively. It is shown that a product of these non-commutative matrices can be represented in terms of the discrepancy norm of the sequence of the corresponding signs,  $-1$  and  $+1$ , respectively. This leads to the intuitive *Lost Walker Lemma*, which immediately provides the solution.

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## *Chan & Vese segmentation method*

*Ayadi Chouikhi-SCCH-KVS*

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### **Abstract:**

During the past several years, image segmentation techniques have been developed and extensively used in a variety of applications as an important tool to extract objects of interest. Usually image segmentation is an initial and vital step in a series of processes aimed at overall image understanding. In an image processing process, we need to divide the image into a number of significant areas and objects we are interested in using image segmentation technology.

Segmentation is the process of partitioning an image into a set of distinct regions, which are different in some important qualitative or quantitative way. This is a critical intermediate step in all high level image processing and computer vision tasks.

In the past decades, many image segmentation algorithms and tools have been developed and are used to process images in different domains, such as pictures taken indoors and outdoors, aerial images, medical images, and videos. Particularly, active contour methods for image segmentation have attracted tremendous interest in the computer vision community in recent years.

Active contour models are popular in the regard. Chan and Vese [1] proposed an active contour without edges scheme based on the classical work of Mumford and Shah [2] variational energy minimization model.

Compared to classical image segmentation methods, such as region splitting/merging, and pixel clustering, active contour methods are more robust by considering well-defined, comprehensive segmentation cost functions and seeking their globally optimal solutions.

Nowadays, added to material use, image segmentation technique has been widely used in object detection and recognition, image editing, image compression, and image database search. For example, segmentation is used as a pre-processing section of object recognition, i.e. face, iris, finger-print recognition etc., to locate or detect the target objects. In addition, in traffic, meteorological, military and medical area, image segmentation is also becoming a vital technique.

This model is used widely in the medical imaging field, especially for the segmentation of the MRI scans of the brain, heart & trachea [3].

**Keywords:** Segmentation, Chan-Vese, level sets [7], Gray, Color/ Texture Image Segmentation.

## CHAN-VESE MODEL FOR GRAY IMAGES: THE FITTING ENERGY FUNCTIONAL

Image gray feature is based on the two properties of gray pixel value: Discontinuity and similarity. Pixels within the region generally have gray similarity.

The Chan–Vese model (CV) is a specific case of the Mumford–Shah problem [2] which solves the minimization of (1) by minimizing the following energy functional:

$$\inf_{c_1, c_2, C} F(c_1, c_2, C) = \mu * Length(C) + \nu * Area(inside(C)) \quad (1)$$

$$+ \lambda_1 * \int_{inside(C)} |u_0 - c_1|^2 dx dy + \lambda_2 * \int_{outside(C)} |u_0 - c_2|^2 dx dy$$

where  $c_1$  and  $c_2$  are two constants which are the average intensities of  $u_0$  inside and outside the contour, respectively.  $\mu$ ,  $\nu$ ,  $\lambda_1$  and  $\lambda_2$  are positive constants,

- The first term of equation (1) controls regularity by penalizing the length of the contour  $C$ .
  - The second term penalizes the enclosed area of  $C$  to control its size.
  - The last two terms penalize discrepancy between the piecewise constant model  $u_0$  and the input image.
- “ $\mu$ ” adjusts the length penalty which balances between fitting the input image more accurately (smaller “ $\mu$ ”) vs. producing a smoother boundary (larger “ $\mu$ ”)
  - “ $\nu$ ” sets the penalty for the area inside  $C$ . When  $\nu$  is large, the object is supposed to be small. And when it is small, the object gets larger.
  - “ $\lambda_1$ ” and “ $\lambda_2$ ” are two regulation parameters for the force pointing inside and the force pointed outside. They control the fitting within each segment. Usually fixing  $\lambda_1 = \lambda_2 = 1$ .

In the paper: Active contours without edges [1], the last two terms can be in general interpreted as two forces.

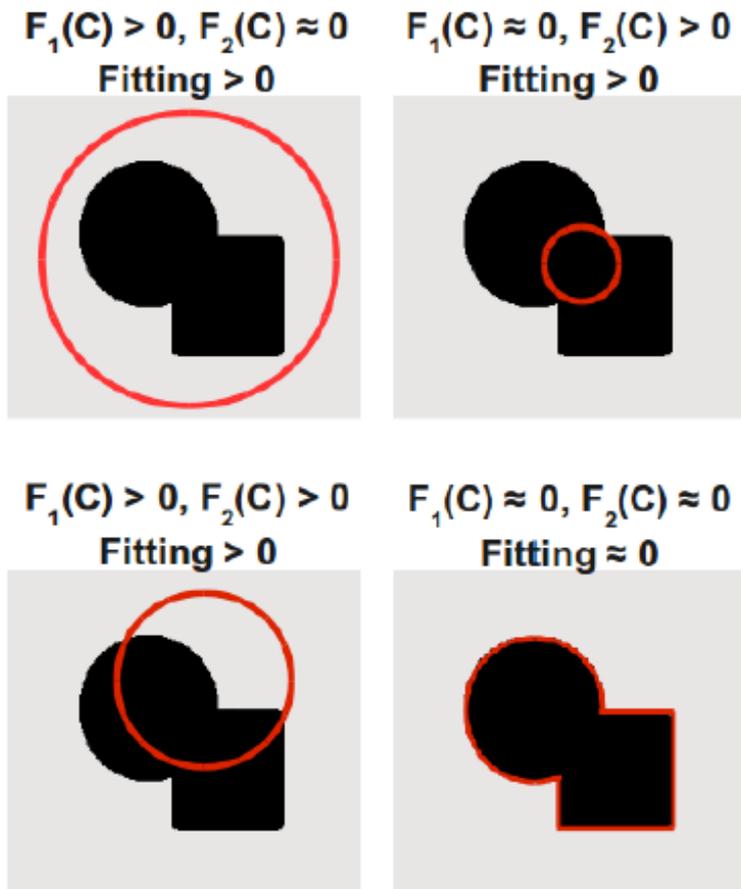
$$F_1(C) + F_2(C) = \lambda_1 * \int_{inside(C)} |u_0 - c_1|^2 dx dy + \lambda_2 * \int_{outside(C)} |u_0 - c_2|^2 dx dy$$

- The first term can be seen as a force to *shrink* the contour.
- The second term as a force to *expand* the contour.

These two forces get balanced when the contour reaches the boundary of the interested object. In another word, the contour  $C$  is equal to the boundary of the objects. Basically, if we swap  $c_1$  and  $c_2$ , the shrinking and expansion is also swapped. These shrinking / expansion operations can be defined at each point of the contour according to the constraints of the image.

In order to understand what is going on with this idea, we can see some figures from the original paper [1].

For example, let's see the following four cases. Simply define everything in black region to -1 and everything in gray region to 1. And here  $c_1$  and  $c_2$  could be interpreted to be the mean value of everything inside of the contour  $C$  and the mean value of everything outside of the contour  $C$ , respectively. Here  $U_0$  stands for the entire image.



**Figure 1:** different positions of the curve. The “fitting energy” is minimized only for the case when the curve is on the boundary of the object (source [1]).

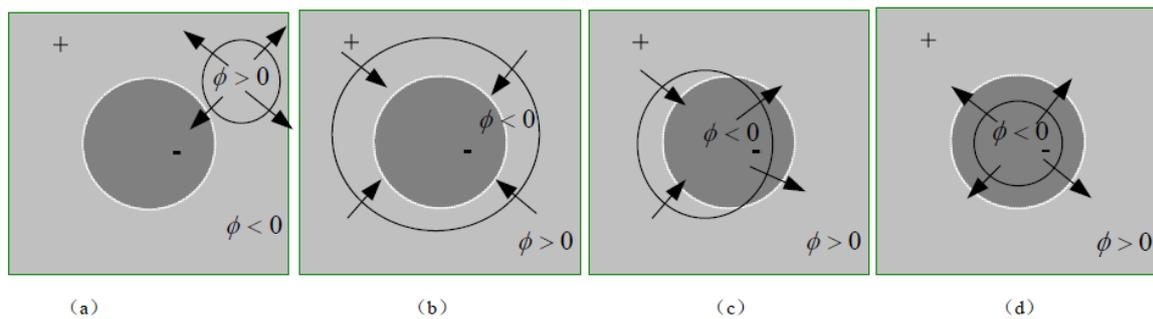
For example if we take the first case (top left):

The initial contour covers the whole object (-1) and some gray region (+1).

Thus  $0 < c_1 < 1$  and  $c_2 = 1$ . Notice the integral above is only respect to inside region of  $u_0$  or outside region. Clearly, if we use everything outside of the contour to minus  $c_2$ , we will get zero. Hence the second term  $F_2 = 0$ . Because  $0 < c_1 < 1$ , when we use everything inside of the contour to minus  $c_1$  and find the sum of the squares as the formula showed, we will reach some big positive number. So we will get  $F_1 > 0$ . Now  $F_1 > 0$  but  $F_2 = 0$ . Therefore, the contour will shrink itself in the next step.

Finally, when our contour  $C$  reaches the boundary of the object,  $F_1 = 0$  and  $F_2 = 0$ . As a result, the contour  $C$  reaches its equilibrium. Hence we find the contour of the object and thus we could get its segmentation.

The figure below can show more clearly the different directions taken by the curve while evolving.



**Figure 2:** Evolution of the curve in the image. Shrinking and/or expansion operations according to the constraints of the image following the curve's normal direction are taken by the curve when evolving.  $\phi$  is the level set function. "-" and "+" define the sign of the level set function.

Note that this method can be extended to segmentation using other image features as color and texture [5], [6]. Besides, another extension of this method is the multiphase-segmentation, which permits to distinguish between more than two regions in one image [4].

## Conclusion

The Chan-Vese algorithm for image segmentation shows that it is effective on a wide variety of images. It is especially useful in cases where an edge-based segmentation algorithm will not suffice, since it relies on global properties (gray level intensities, contour lengths, region areas) rather than local properties such as gradients. This means that it can deal gracefully with noisy images, blurry images, and images where

the foreground region has a complicated topology (multiple holes, disconnected regions, etc).

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# Dynamic Inclusion of New Event Types in Visual Inspection using Evolving Classifiers

Edwin Lughofer

November 20, 2014

## Abstract

In this talk, we are dealing with the automatic inclusion of new event types in visual inspection systems. Within the context of image classification for recognizing "OK" and "not OK" parts, a certain event can be directly associated with a class, as events are usually independent and disjoint from each other. In this sense, we are dealing with the problem of integrating a new class into the image classifier on-the-fly, once specified on-line by an operator. We are using evolving fuzzy classifiers (EFC), which are relying on fuzzy rule bases and are able to adapt their structure and update their parameters in incremental manner. The novel methodological aspects lie 1.) in appropriate structural changes in the EFC whenever a new class appears and 2.) in the estimation of the expected change in classifier accuracy on the older classes seen before, which is based on an analysis of the expected change in the classifier's decision boundaries. The second point is an important aspect for operators, as they are already familiar to work with established classifiers that have some accuracy in classification. The new concepts will be evaluated on a real-world visual inspection scenario, where the main task is to classify several event types which may occur on micro-fluidic chips and may lead to the deterioration of their quality. The evaluation will be based on two image streams recorded at the inspection system on-line, containing several event types and representing the real production order.

**Keywords:** visual inspection, new event types, integration of new classes on-the-fly, evolving (fuzzy) classifiers, expected change in classifier's accuracy